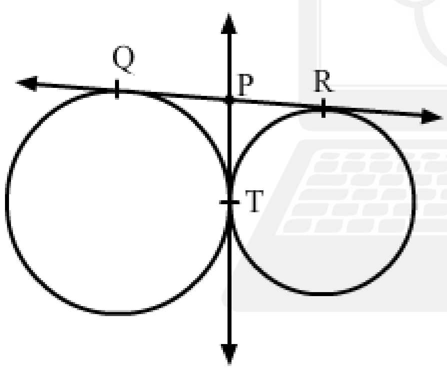
CBSE Class X Math Paper 2014

Q1. The first three terms of an AP are 3y-1, 3y+5 and 5y+1, respectively.

We need to find the value of y.
We know that if a, b and c are in AP, then:
$b - a = c - b \Rightarrow 2b = a + c$

$\therefore 2(3y+5) = 3y - 1 + 5y + 1$
$\Rightarrow 6y + 10 = 8y$
$\Rightarrow 10 = 8y - 6y$
$\Rightarrow 2y = 10$
$\Rightarrow y = 5$
Hence the correct option is C.

Q2.



It is known that the length of the tangents drawn from an external point to a circle is equal.

$\therefore QP = PT = 3.8$ cm ... (1)

$PR = PT = 3.8$ cm ... (2)

From equations (1) and (2), we get:

$QP = PR = 3.8$ cm

Now, $QR = QP + PR$
$= 3.8 \text{ cm} + 3.8 \text{ cm}$
$= 7.6$ cm

Hence, the correct option is B.

Q3. Given: $\angle QPR = 46°$
PQ and PR are tangents.
Therefore, the radius drawn to these tangents will be perpendicular to the tangents.
So, we have $OQ \perp PQ$ and $OR \perp RP$.
$\Rightarrow \angle OQP = \angle ORP = 90°$
So, in quadrilateral PQOR, we have
$\angle OQP + \angle QPR + \angle PRO + \angle ROQ = 360°$
$\Rightarrow 90° + 46° + 90° + \angle ROQ = 360°$
$\Rightarrow \angle ROQ = 360° - 226° = 134°$

Hence, the correct option is B.

Q4.

In the figure, MN is the length of the ladder, which is placed against the wall AB and makes an angle of 60° with the ground.

The foot of the ladder is at N, which is 2 m away from the wall.

$\therefore$ BN = 2 m

In right-angled triangle MNB:

$\cos 60° = \frac{BN}{MN} = \frac{2\text{ m}}{MN}$

$\Rightarrow \frac{1}{2} = \frac{2\text{ m}}{MN}$

$\Rightarrow$MN = 4 m

Therefore, the length of the ladder is 4 m.

Hence, the correct option is D

Q5. Possible outcomes on rolling the two dice are given below:

{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Total number of outcomes = 36

Favourable outcomes are given below:

{(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)}

Total number of favourable outcomes = 9

∴ Probability of getting an even number on both dice =

$$\frac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{9}{36} = \frac{1}{4}$$

Hence, the correct option is D.

Q6. Total number of possible outcomes = 30

Prime numbers between 1 to 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

Total number of favourable outcomes = 10

∴ Probability of selecting a prime number from 1 to 30

$$= \frac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{10}{30} = \frac{1}{3}$$

Hence, the correct option is C.

Q7. It is given that the three points A(x, 2), B(−3, −4) and C(7, −5) are collinear.

∴ Area of ΔABC = 0

$\Rightarrow \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right] = 0$

Here, $x_1 = x$, $y_1 = 2$, $x_2 = -3$, $y_2 = -4$, and $x_3 = 7$, $y_3 = -5$

$\Rightarrow x[-4 - (-5)] - 3(-5 - 2) + 7[2 - (-4)] = 0$

$\Rightarrow x(-4 + 5) - 3(-5 - 2) + 7(2 + 4) = 0$

$\Rightarrow x - 3 \times (-7) + 7 \times 6 = 0$

$\Rightarrow x + 21 + 42 = 0$

$\Rightarrow x + 63 = 0$

$\Rightarrow x = -63$

Thus, the value of x is − 63.
Hence, the correct option is A.

Q8. Let r and h be the radius and the height of the cylinder, respectively.
Given: Diameter of the cylinder = 4 cm
∴ Radius of the cylinder, r = 2 cm
Height of the cylinder, h = 45 cm
Volume of the solid cylinder = $\pi r^2 h = \pi \times (2)^2 \times 45$ cm^3 = 180π cm^3

Suppose the radius of each sphere be R cm.
Diameter of the sphere = 6 cm
∴ Radius of the sphere, R = 3 cm

Let n be the number of solids formed by melting the solid metallic cylinder.

∴ n × Volume of the solid spheres = Volume of the solid cylinder

$\Rightarrow n \times \frac{4}{3}\pi R^3 = 180\pi$

$\Rightarrow n \times \frac{4}{3}\pi 3^3 = 180\pi$

$$\Rightarrow n = \frac{180 \times 3}{4 \times 27} = 5$$

Thus, the number of solid spheres that can be formed is 5.

Hence, the correct option is B.

Q9. We have: $2x^2 + ax - a^2 = 0$

Comparing the given equation with the standard quadratic equation ($ax^2 + bx + c = 0$), we get

a =2, b = a and c =$-a^2$

Using the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, we get:

$$x = \frac{-a \pm \sqrt{a^2 - 4 \times 2 \times (-a)^2}}{2 \times 2}$$

$$= \frac{-a \pm \sqrt{9a^2}}{4}$$

$$= \frac{-a \pm 3a}{4}$$

$\Rightarrow x = \frac{-a+3a}{4} = \frac{a}{2}$ or $x = \frac{a-3a}{4} = -a$

So, the solutions of the given quadratic equation are $x = \frac{a}{2}$ or $x = -a$.

Q10. Let a be the first term and d be the common difference.

Given: a = 5

$T_n = 45$

$S_n = 400$

We know:

$T_n = a + (n - 1) d$

$\Rightarrow 45 = 5 + (n - 1)d$

$\Rightarrow 40 = (n - 1) d$ ………….(1)

And $S_n = \frac{n}{2} a + T_n$

$\Rightarrow 400 = \frac{n}{2}(5+45)$

$\Rightarrow \frac{n}{2} = \frac{400}{50}$

$\Rightarrow n = 2 \times 8 = 16$

On substituting n = 16 in (1), we get:

$40 = (16-1)d$

$\Rightarrow 40 = (15)d$

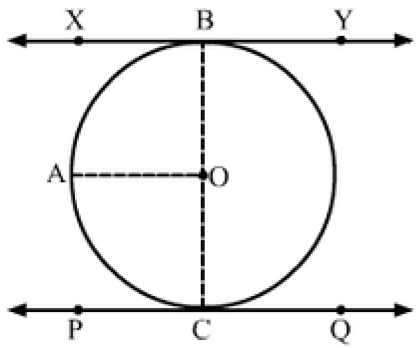
$\Rightarrow d = \frac{40}{15} = \frac{8}{3}$

Thus, the common difference is $\frac{8}{3}$.

Q11. Let XBY and PCQ be two parallel tangents to a circle with centre O.

Construction: Join OB and OC.

Draw OA||XY



Now, XB ||AO

$\Rightarrow \angle XBO + \angle AOB = 180°$ (sum of adjacent interior angles is 180°)

Now, $\angle XBO = 90°$ (A tangent to a circle is perpendicular to the radius through the point of contact)

$\Rightarrow 90° + \angle AOB = 180°$

$\Rightarrow \angle AOB = 180° - 90° = 180°$

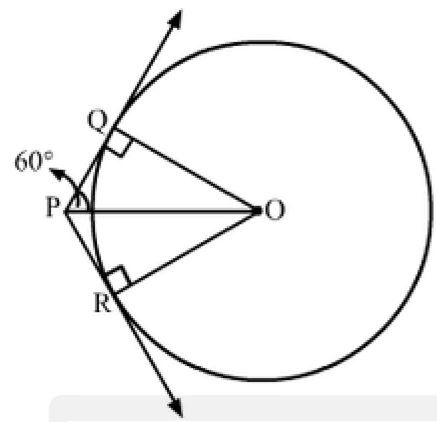
Similarly, $\angle AOC = 90°$

$\angle AOB + \angle AOC = 90° + 90° = 180°$

Hence, BOC is a straight line passing through O.

Thus, the line segment joining the points of contact of two parallel tangents of a circle passes through its centre.

Q12. Let us draw the circle with extent point P and two tangents PQ and PR.



We know that the radius is perpendicular to the tangent at the point of contact.

∴∠OQP = 90°

We also know that the tangents drawn to a circle from an external point are equally inclined to the joining the centre to that point.

∴∠QPO = 60°

Now, in ΔQPO:

Cos 60°= $\frac{PQ}{PO}$

$\Rightarrow \frac{1}{2} = \frac{PQ}{PO}$

⇒2PQ = PO

Q13. Rahim tosses two coins simultaneously. The sample space of the experiment is {HH, HT, TH, and TT}.

Total number of outcomes = 4

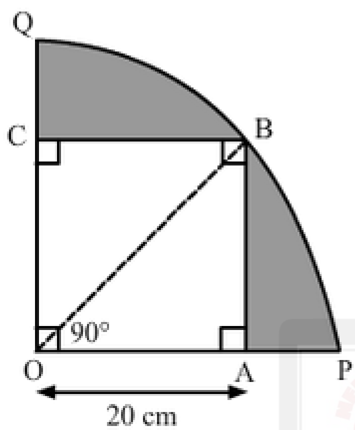
Outcomes in favour of getting at least one tail on tossing the two coins = {HT, TH, TT}

Number of outcomes in favour of getting at least one tail = 3

∴ Probability of getting at least one tail on tossing the two coins

$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{3}{4}$

Q14. Let us join OB.



In ΔOAB:

$OB^2 = OA^2 + AB^2 = (20)^2 + (20)^2 = 2 \times (20)^2$

$\Rightarrow OB = 20\sqrt{2}$

Radius of the circle, $r = 20\sqrt{2}$ cm

Area of quadrant OPBQ $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{90°}{360°} \times 3.14 \times (20\sqrt{2})^2 cm^2$

$= \frac{1}{4} \times 3.14 \times 800$ cm2

$= 628\ cm^2$

Area of square OABC = $(Side)^2 = (20)^2\ cm^2 = 400\ cm^2$

∴ Area of the shaded region = Area of quadrant OPBQ – Area of square OABC

$= (628 - 400)\ cm^2$

$= 228\ cm^2$

Q 15. Given equation:

$$\frac{4}{x} - 3 = \frac{5}{2x+3} \text{ ; } x \neq 0, -\frac{3}{2}$$

$$\frac{4}{x} - 3 = \frac{5}{2x+3}$$

$$\Rightarrow \frac{4-3x}{x} = \frac{5}{2x+3}$$

$\Rightarrow$ (4 -3x) (2x+3) = 5x

$\Rightarrow -6x^2 + 8x - 9x + 12 = 5x$

$\Rightarrow 6x^2 + 6x - 12 = 0$

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow x^2 + 2x - x - 2 = 0$

$\Rightarrow (x+2)(x-1) = 0$

$\Rightarrow (x+2) = 0$ $(x-1) = 0$

$\Rightarrow x = -2$ or $x = 1$

Thus, the solution of the given equation is -2 or 1.

Q16. Let a be the first term and d be the common difference of the given A.P.

Given:

$a_7 = \frac{1}{9}$

$a_9 = \frac{1}{7}$

$a_7 = a + (7-1)\, d = \frac{1}{9}$

$\Rightarrow a + 6d = \frac{1}{9}$ .... (1)

$a_9 = a + (9-1)\, d = \frac{1}{7}$

$\Rightarrow a + 8d = \frac{1}{7}$ .... (2)

Subtracting equation (1) from (2), we get:

$2d = \frac{2}{63}$

$\Rightarrow d = \frac{1}{63}$

Putting $d = \frac{1}{63}$ in equation (1), we get:

$a + (6 \times \frac{1}{63}) = \frac{1}{9}$

$\Rightarrow a = \frac{1}{63}$

$\therefore a_{63} = a + (63 - 1)\, d = \frac{1}{63} + 62\,(\frac{1}{63}) = \frac{63}{63} = 1$

Thus, the 63rd term of the given A.P. is 1.

Q17. Follow the given steps to construct the figure.

Step 1

Draw a line BC of 8 cm length.

Step 2

Draw BX perpendicular to BC.

Step 3

Mark an arc at the distance of 6 cm on BX. Mark it as A.

Step 4

Join A and C. Thus, ΔABC is the required triangle.

Step 5

With B as the centre, draw an arc on AC.

Step 6

Draw the bisector of this arc and join it with B. Thus, BD is perpendicular to AC.

Step 7

Now, draw the perpendicular bisector of BD and CD. Take the point of intersection as O.

Step 8

With O as the centre and OB as the radius, draw a circle passing through points B, C and D.

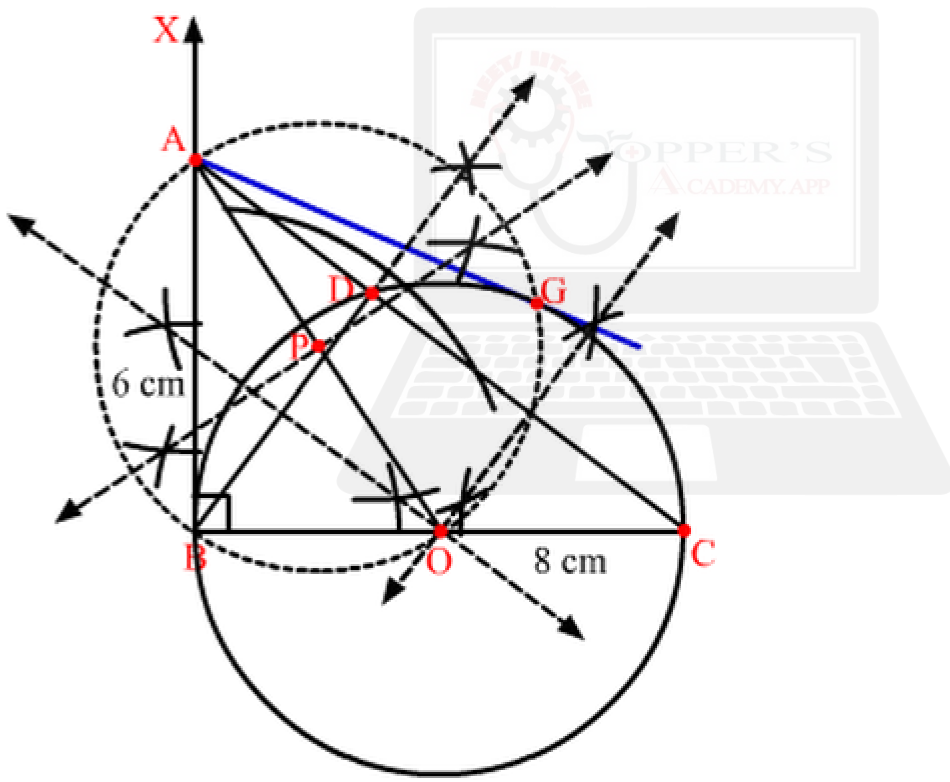
Step 9

Join A and O and bisect it Let P be the midpoint of AO.

Step 10

Taking P as the centre and PO as its radius, draw a circle which will intersect the circle at point B and G. Join A and G.

Here, AB and AG are the required tangents to the circle from A.



Q 18. The given points are A (0, 2), B (3, p) and C (p, 5).

It is given that A is equidistant from B and C.

∴ AB = AC

$\Rightarrow$ $AB^2 = AC^2$

$\Rightarrow$ $(3\text{-}0)^2 + (p\text{-}2)^2 = (p\text{-}0)^2 + (5\text{-}2)^2$

$\Rightarrow$ $9+p^2+ 4\text{-} 4p=p^2+9$

$\Rightarrow$ $4 - 4p = 0$

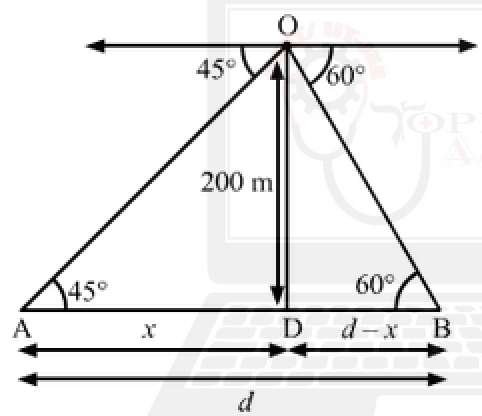
$\Rightarrow$ $4p = 4$

$\Rightarrow$ $p = 1$

Thus, the value of p is 1.

Length of AB = $\sqrt{(3-0)^2+(1-2)^2} = \sqrt{3^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$units.

Q19. Let d be the distance between the two ships. Suppose the distance of one of the ships from the light house is X meters, then the distance of the other ship from the light house is (d-x) meter.



In right –angled ΔADO, we have.

$\tan 45° = \frac{OD}{AD} = \frac{200}{X}$

$\Rightarrow 1 = \frac{200}{X}$

$\Rightarrow x = 200$ … (1)

In right-angled Δ BDO, we have

$\tan 60° = \frac{OD}{BD} = \frac{200}{d-x}$

$\Rightarrow \sqrt{3} = \frac{200}{d-x}$

$\Rightarrow d - x = \frac{200}{\sqrt{3}}$

Putting x=200. We have:

$d - 200 = \frac{200}{\sqrt{3}}$

$d = \frac{200}{\sqrt{3}} + 200$

$\Rightarrow d = 200(\frac{\sqrt{3}+1}{\sqrt{3}})$

$\Rightarrow d = 200 \times 1.58$

$\Rightarrow d = 316$ (approx.)

Thus, the distance between two ships is approximately 316 m.

Q20. The given points are A (-2, 1), B (a, b) and C (4-1).

Since the given points are collinear, the area of the triangle ABC is 0.

$\Rightarrow \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y1 - y2)] = 0$

Here, $x_1=-2, y_1=1, x_2=a, y_2=b$ and $x_3=4$, $y_3=-1$

$\therefore \frac{1}{2}[-2(b+1)+a(-1-1)+4(1-b) = 0$

$\Rightarrow -2b -2-2a+4-4b=0$

$\Rightarrow 2a+6b=2$

$\Rightarrow a+3b=1$ ....(1)

Given :

a-b= 1 ....(2)

Subtracting equation (1) from (2) we get:

4b =0

$\Rightarrow b=0$

Subtracting b= 0 in (2), we get:

a= 1

Thus, the values of a and b are 1 and 0, respectively.

Q21. It is given that ABC is an equilateral triangle of side 12 cm.

Construction:

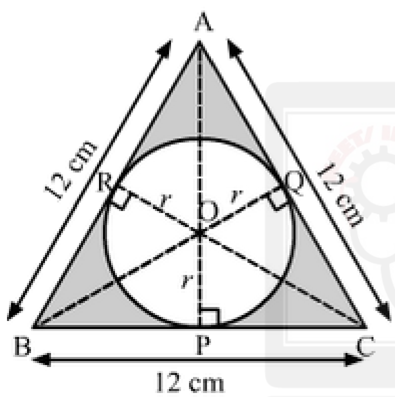
Join OA, OB and OC.

Draw.

OP $\perp$ BC

OQ $\perp$ AC

OR $\perp$ AB



Let the radius of the circle be r cm.

Area of ΔAOB +Area of ΔBOC + Area of ΔAOC = Area of ΔABC

$\Rightarrow \frac{1}{2} \times AB \times OR + \frac{1}{2} \times BC \times OP + \frac{1}{2} \times AC \times OQ = \frac{\sqrt{3}}{4} \times (\text{Side})^2$

$\Rightarrow \frac{1}{2} \times 12 \times r + \frac{1}{2} \times 12 \times r + \frac{1}{2} \times 12 \times r = \frac{\sqrt{3}}{4} \times (12)^2$

$\Rightarrow 3 \times \frac{1}{2} \times 12 \times r = \frac{\sqrt{3}}{4} \times 12 \times 12$

$\Rightarrow r = 2\sqrt{3} = 2 \times 1.73 = 3.46$

Therefore the radius of the inscribed circle is 3.46 cm.

Now, area of the shaded region = Area of ΔABC – Area of the inscribed circle

$= [\frac{\sqrt{3}}{4} \times (12)^2 - \pi\ (2\sqrt{3})^2]\ \text{cm}^2$

$= [36\sqrt{3} - 12\pi]\ \text{cm}^2$

$= [36 \times 1.73 - 12 \times 3.14]$ cm^2

$= [62.28 - 37.68]$ cm^2

$= 24.6$ cm^2

Therefore, the area of the shaded region is 24.6 cm2.

Q22. Radius of Semicircle PSR = $\frac{1}{2} \times 10$ cm =5 cm

Radius of Semicircle RTQ = $\frac{1}{2} \times 3 = 1.5$ cm

Radius of semicircle PAQ = $\frac{1}{2} \times 7$ cm = 3.5 cm

Perimeter of the shaded region = Circumference of semicircle PSR + Circumference of semicircle RTQ + Circumference of semicircle PAQ

$= [\frac{1}{2} \times 2\pi(5) + \frac{1}{2} \times 2\pi\ (1.5) + \frac{1}{2} \times 2\pi(3.5)]$ cm

$= \pi\ (5+1.5+3.5)$ cm

$=3.14 \times 10$ cm

$= 31.4$ cm

Q23. For the given tank.

Diameter = 10 m

Radius, R = 5m

Depth, H =2m

Internal radius of the pipe = r $= \frac{20}{2}$ cm =10 cm $= \frac{1}{10}$ m

Rate of flow of water = v = 4 km/h = 4000 m/h

Let t be the time taken to fill the tank.

So, the water flown through the pipe in t hours will equal to the volume of the tank

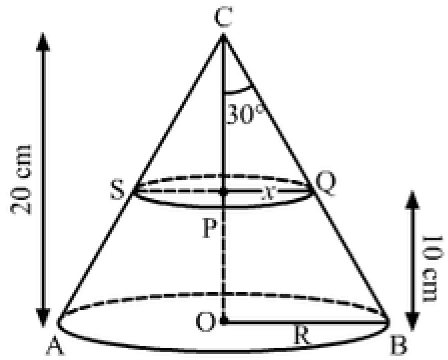
$\therefore \pi r^2 \times v \times t = \Pi R^2 H$

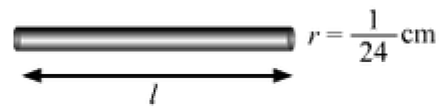
$\Rightarrow (\frac{1}{10})^2 \times 4000 \times t = (5)^2 \times 2$

$\Rightarrow t = \frac{25 \times 2 \times 100}{4000} = 1\frac{1}{4}$

Hence, the time taken is $1\frac{1}{4}$ hours.

Q 24.





Let ACB be the cone whose vertical angle $\angle ACB = 60°$. Let R and x be the radii of the lower and upper end of the frustum.

Here, height of the cone, $OC = 20 \text{ cm} = H$

Height $CP = h = 10$ cm

Let us consider P as the mid-Point of OC.

After cutting the cone into two parts through P.

$OP = \frac{20}{2} = 10$ cm

Also, $\angle ACO$ and $\angle OCB = \frac{1}{2} \times 60° = 30°$

After cutting cone CQS from cone CBA, the remaining solid obtained is a frustum.

Now, in triangle CPQ:

$\tan 30° = \frac{x}{10}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{10}$

$\Rightarrow x = \frac{10}{\sqrt{3}}$ cm

In triangle COB:

$\text{Tan } 30° = \frac{R}{CO}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{R}{20}$

$\Rightarrow R = \frac{20}{\sqrt{3}}$ cm

Volume of the frustum, $V = \frac{1}{3}\pi\ (R^2H - x^2h)$

$\Rightarrow V = \frac{1}{3}\pi\left(\left(\frac{20}{\sqrt{3}}\right)^2 .20 - \left(\frac{10}{\sqrt{3}}\right)^2 .10\right)$

$= \frac{1}{3}\pi\ \left(\frac{8000}{3} - \frac{1000}{3}\right)$

$= \frac{1}{3}\pi\ \left(\frac{7000}{3}\right)$

$= \frac{1}{9}\pi \times 7000$

$= \frac{7000}{9}\pi$

The volumes of the frustum and the wire formed are equal.

$\pi \times \left(\frac{1}{24}\right)^2 \times l = \frac{7000}{9}\ \pi$ [Volume of wire = $\pi r^2 h$]

$\Rightarrow l = \frac{7000}{9} \times 24 \times 24$

$\Rightarrow l = 448000$ cm $= 4480$ m

Hence, the length of the wire is 4480 m.

Q25. Let the two natural numbers be X and Y such that $x > y$.

Given:

Difference between the natural numbers = 5

$\therefore$ X - Y =5 .....(1)

Difference of their reciprocals = $\frac{1}{10}$ (given)

$\frac{1}{y} - \frac{1}{x} = \frac{1}{10}$

$\Rightarrow \frac{x-y}{xy} = \frac{1}{10}$

$\Rightarrow \frac{5}{xy} = \frac{1}{10}$

$\Rightarrow xy = 50$ … (ii)

Putting the value of x from equation (i) in equation (ii), we get

$(y+5)\ y = 50$

$\Rightarrow y^2 + 5y - 50 = 0$

$\Rightarrow y^2 + 10y - 5y - 50 = 0$

$\Rightarrow y\ (y+10) - 5\ (y+10) = 0$

$\Rightarrow (y - 5)\ (y + 10) = 0$

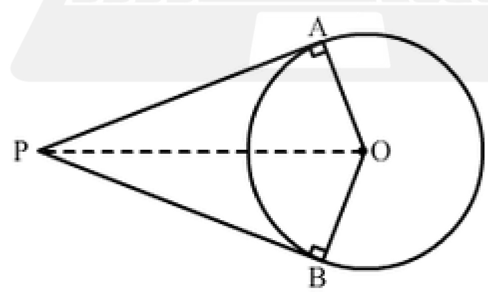
$\Rightarrow y = 5$ or -10

As y is a natural number, therefore y = 5

Other natural number = y + 5 = 5 + 5 = 10

Thus, the two natural numbers are 5 and 10.

Q26. Let AP and BP be the two tangents to the circle with centre O.



To Prove : AP = BP

Proof:

In ΔAOP and ΔBOP

OA = OB (radii of the same circle)

$\angle OAP = \angle OBP = 90°$ (since tangent at any point of a circle is perpendicular to the radius through the point of contact)

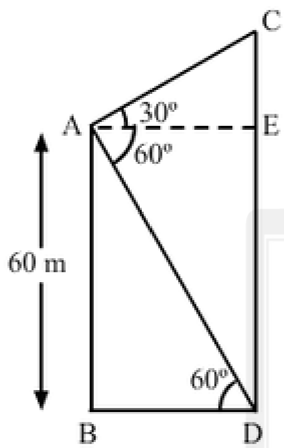
OP = OP (common)

∴ ΔAOP ≅ Δ OBP (by R.H.S. congruence criterion)

∴ AP = BP (corresponding parts of congruent triangles)

Hence the length of the tangents drawn from an external point to a circle are equal.

Q27. Let AB be the building and CD be the tower.



In right ΔABD.

$\frac{AB}{BD}$ = tan 60°

$\Rightarrow \frac{60}{BD} = \sqrt{3}$

$\Rightarrow BD = \frac{60}{\sqrt{3}}$

$\Rightarrow BD = 20\sqrt{3}$

In right ΔACE:

$\frac{CE}{AE}$ = tan 30°

$\Rightarrow \frac{CE}{BD} = \frac{1}{\sqrt{3}}$ (∴ AE = BD)

$\Rightarrow CE = \frac{20\sqrt{3}}{\sqrt{3}} = 20$

Height of the tower = CE + ED = CE + AB = 20 m + 60 m = 80 m

Difference between the heights of the tower and the building = 80 m – 60m = 20 m

Distance between the tower and the building = BD = $20\sqrt{3}$ m.

Q28. Total number of cards = 49

(1)

Total number of outcomes = 49

The odd numbers form 1 to 49 are 1, 3,5,7,9,11,13,15,17, 19,21, 23, 2527,29,31,33,35, 37,

39, 41, 43, 45, 47 and 49.

Total number of favourable outcomes = 25

$\therefore$ Required probability = $\frac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{25}{49}$

(ii)

Total number of outcomes = 49

The number 5,10,15,20,25,30,35,40 and 45 multiples of 5.

The number of favourable outcomes = 9

$\therefore$ Required probability = $\frac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{9}{49}$

(iii) Total number of outcomes = 49

The number 1, 4, 9, 16, 25, 36 and 49 are perfect squares.

Total number of favourable outcomes = 7

$\therefore$ Required probability = $\frac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{7}{49} = \frac{1}{7}$

(iv)

Total number of outcomes = 49

We know that there is only one even prime number which is 2

Total number of favourable outcomes = 1

$\therefore$ Required probability = $\frac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{1}{49}$

Q29. Let the Point P (x, 2) divide the line segment joining the points A (12, 5) and B (4, -3) in the ratio k: 1

Then, the coordinates of P are ($\frac{4k+12}{k+1}$, $\frac{-3k+5}{k+1}$)

Now, the coordinates of P are (x,2)

$\therefore \frac{4k+12}{k+1}$= x and $\frac{-3k+5}{k+1}$ = 2

$\frac{-3k+5}{k+1}$ = 2

$\Rightarrow - 3k+ 5 = 2k+2$

$\Rightarrow$ 5k = 3

$\Rightarrow k = \frac{3}{5}$

Substituting k = $\frac{3}{5}$ in $\frac{4k+12}{k+1}$= x, we get

$X = \frac{4\times\frac{3}{5}+12}{\frac{3}{5}+1}$

$\Rightarrow x = \frac{12+60}{3+5}$

$\Rightarrow x = \frac{72}{8}$

$\Rightarrow$ x = 9

Thus, the value of x is 9

Also, the point P divides the line segment joining the points A (12, 5) and (4, -3) in the ratio $\frac{3}{5}$: 1, i.e. 3:5.

Q30. Given quadratic equation:

$(k + 4)x^2 + (k + 1)x + 1=0$

Since the given quadratic equation has equal roots, Its discriminant should be zero.

$\therefore$ D = 0

$\Rightarrow (k+1)^2 - 4 \times (k+4) \times 1 = 0$

$\Rightarrow k^2+2k+1- 4k - 16=0$

$\Rightarrow k^2 - 2k -15 =0$

$\Rightarrow k^2 - 5k + 3k - 15 = 0$

$\Rightarrow (k-5)(k+3) = 0$

$\Rightarrow k-5 = 0$ or $k+3 = 0$

$\Rightarrow k = 5$ or -3

Thus, the values of k are 5 and -3

For k = 5

$(k+4)x^2 + (k+1)x + 1 = 0$

$\Rightarrow 9x^2 + 6x + 1 = 0$

$\Rightarrow (3x)^2 + 2(3x) + 1 = 0$

$\Rightarrow (3x+1)^2 = 0$

$\Rightarrow x = -\frac{1}{3}, -\frac{1}{3}$

For k = -3

$(k+4)x^2 + (k+1)x + 1 = 0$

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow (x-1)^2 = 0$

$\Rightarrow x = 1, 1$

Thus, the equal root of the given quadratic equation is either 1 or $-\frac{1}{3}$.

Q31 Let a and d be the first term and the common difference of an A. P. respectively.

n^{th} term of an A. P, $a_n = a + (n-1)d$

Sum of n terms of an A. P , $S_n = \frac{n}{2}[2a+(n-1)d]$

We have:

Sum of the first 10 terms = $\frac{10}{2}[2a + 9d]$

$\Rightarrow 210 = 5[2a+9d]$

$\Rightarrow 42 = 2a+9d$ .... (1)

15^{th} term from the last = $(50-15+1)^{th}$ = 36^{th} term from the beginning

Now, $a_{36} = a + 35d$

$\therefore$ Sum of the last 15 terms = $\frac{15}{2}(2a_{36} + (15-1)d)$

$= \frac{15}{2}[2(a + 35d) + 14d]$

$= 15[a + 35d + 7d]$

$\Rightarrow 2565 = 15[a + 42d]$

$\Rightarrow 171 = a + 42d$ …………(2)

From (1) and (2), we get,

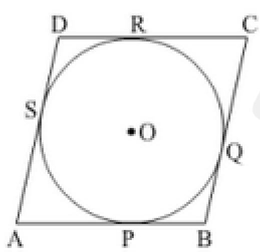
$d = 4$

$a = 3$

So, the A. P. formed is 3, 7, 11, 15 …. and 199.

Q32 Given ABCD be a parallelogram circumscribing a circle with centre O.

To Prove: ABCD is a rhombus.



We know that the tangents drawn to a circle from an exterior point are equal is length.

$\therefore$ AP = AS, BP = BQ, CR = CQ AND DR = DS.

AP + BP + CR + DR = AS + BQ + CQ + DS

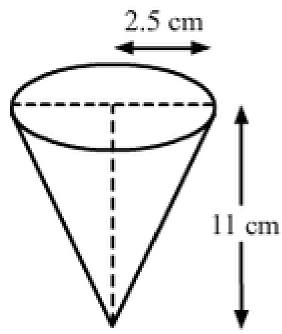
(AP + BP) + (CR + DR) = (AS + DS) + BQ + CQ)

$\therefore$AB + CD = AD + BC OR 2AB = 2BC (since AB = DC and AD = BC)

$\therefore$ AB = BC = DC = AD

Therefore, ABCD is a rhombus.

Q33.



Height (h) of the conical vessel = 11 cm

Radius (r_1) of the conical Vessel = 2.5 cm

Radius (r_2) of the metallic spherical balls = $\frac{0.5}{2}$ = 0.25 cm

Let n be the number of spherical balls =that were dropped in the the vessel.

Volume of the water spilled = Volume of the spherical balls dropped

$\frac{2}{5}$ × Volume of cone = n × Volume of one spherical ball

$\Rightarrow \frac{2}{5} \times \frac{1}{3} \pi r_1^2 h = n \times \frac{4}{3} \pi r_2^3$

$\Rightarrow r_1^2 h = n \times 10 r_2^3$

$\Rightarrow (2.5)^2 \times 11 = n \times 10 \times (0.25)^3$

$\Rightarrow 68.75 = 0.15625n$

$\Rightarrow n = 440$

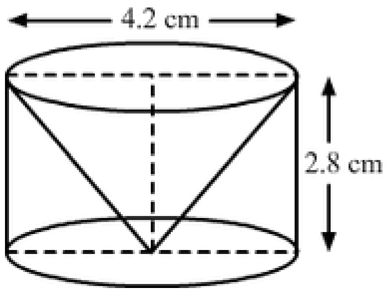
Hence, the number of spherical balls that were dropped in the vessel is 440.

Sushant made the arrangement so that the water that flows out, irrigates the flower beds.

This shows the judicious usage of water.

Q34.

The following figure shows the required cylinder and the conical cavity.



Given

Height (b) of the conical Part = Height (h) of the cylindrical part = 2.8 cm

Diameter of the cylindrical part = Diameter of the conical part = 4.2 cm

∴ Radius ® of the cylindrical part = Radius ® of the conical part = 2.1 cm

Slant height (l) of the conical part $=\sqrt{r^2 + h^2}$

$=\sqrt{(2.1)^2 + (2.8)^2}$ cm

$=\sqrt{4.41 + 7.81}$ cm

$= \sqrt{12.25}$ cm

= 3.5 cm

Total surface area of the remaining solid = Curved surface area of the cylindrical part +Curved surface area of the conical part + Area of the cylindrical base

= 2πrh +πrl +πr^2

$=(2 \times \frac{22}{7} \times 2.1 \times 2.8 + \frac{22}{7} \times 2.1 \times 3.5 + \frac{22}{7} \times 2.1 \times 2.1)$ cm^2

= (36.96 + 23.1 + 13.86) cm^2

=73.92 cm^2

Thus, the total surface area of the remaining solid is 73.92 cm^2